Problems

1. (20 pts) A cube of solid material falls to the bottom of a lake where the water pressure is P. The solid has a Young's modulus of Y_m and a Poisson ratio σ. Using superposition, show that the fractional change in volume is given by,

$$\frac{\Delta V}{V} = -\frac{3(1-2\sigma)}{Y_m} P = -\frac{P}{K}$$

where *K* is called the *bulk modulus*. Remember that to use superposition you must work in the limit of small changes in which stress and strain are proportional. You can ignore terms that are second order in any small changes.

2. (20 pts) Prove the generalized Snell's Law that includes both longitudinal and transverse waves in a solid,

$$\frac{sin\theta_i^P}{c_{P1}} = \frac{\sin\theta_r^S}{c_{S1}} = \frac{\sin\theta_t^P}{c_{P2}} = \frac{\sin\theta_t^S}{c_{S2}} \qquad \theta_i^P = \theta_r^P$$

3. (20 pts) Show that the longitudinal and transverse wave velocities in an isotropic solid are given by the expressions in the notes,

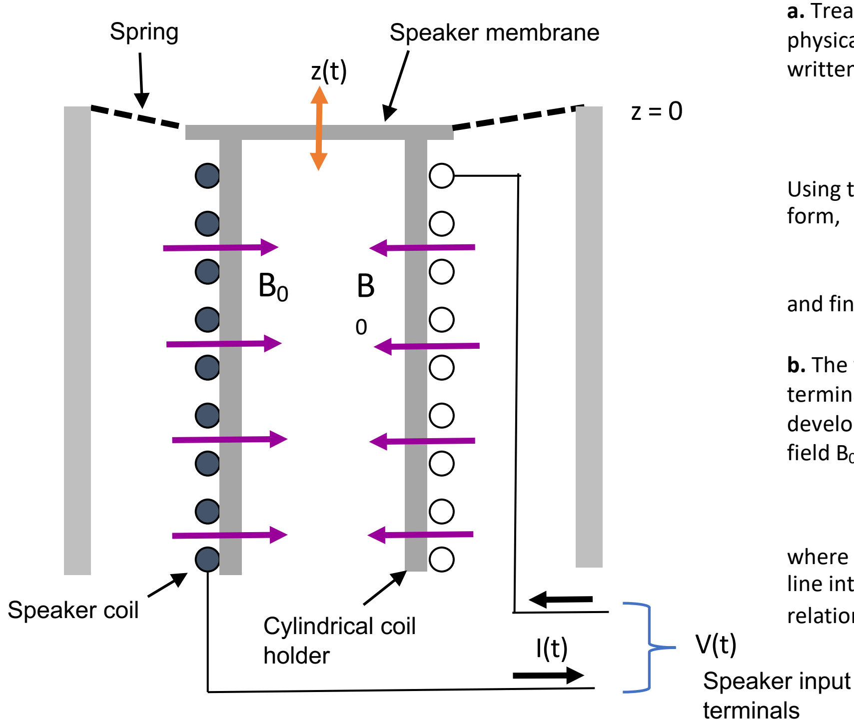
$$c_P = \sqrt{(\lambda + 2\mu)/\varrho} \qquad c_S = \sqrt{\mu/\varrho}$$

4. (10 pts) Consider a sound wave in air whose frequency is 10^3 Hz and whose RMS pressure amplitude *p*' = 0 dBSPL. This is at the threshold of human hearing. Find the peak amplitude of the air displacement u_{peak} where $v_x = \partial u/\partial t$ and v_x is the fluid velocity in the sound wave.

5. (30 pts) The basic elements of a loudspeaker are shown below. The speaker membrane is rigidly attached to a cylindrical speaker coil holder that can move back and forth and produce sound. To make it move, the holder has a coil of wire of total length L attached to it. The coil sits in a static magnetic field B0 that point radially inward as shown. When a current *I*(t) passes through the coil, the resulting Lorentz force causes it to move back and forth, moving the membrane and producing sound. If a voltage generator is attached to the input terminals, the speaker appears like a resonant LC circuit. To find the circuit parameters assume that the coil, its holder and the membrane move rigidly along the z-direction as shown. Treat this assembly as a simple harmonic oscillator with mass M, spring constant *k* and damping constant γ so it has an equation of motion,

$$M\frac{d^2z}{dt^2} + \gamma\frac{dz}{dt} + kz = F_B(t)$$

F(t) is the Lorentz force on the coil due to the current *I*(t) and the static magnetic field B_0 .



a. Treat all quantities as complex phasors in which physical quantities (force, position, current, voltage) are written in the form,

$$I(t) = Re\left(\hat{I}\, e^{i\omega t}\right)$$

Using this approach, write the equation of motion in the form,

$$\hat{z} = g(\omega)\hat{I}$$

and find the complex function $g(\omega)$.

b. The voltage $V(t) = Re\left(\hat{V}\, e^{i\omega t}\right)$ across the speaker terminals can be obtained from the motional EMF developed across the coil as it moves back and forth the field B_0. This is given by,

$$EMF = \oint \vec{v}\; x\; \vec{B}_0\, d\vec{l}$$

where $\vec{v}$ is the velocity of the coil at each point and the line integral is taken around the coil. Using this find a relation between $\hat{V}$ and $\hat{z}$.

c. Show that current and voltage phasors are related by,

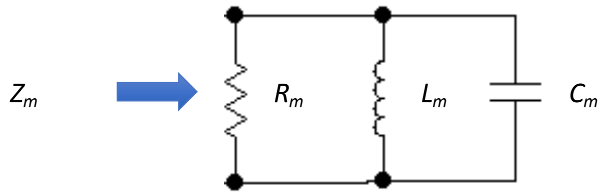
$$\hat{I} = \left(i\omega C_m + \frac{1}{R_m} + \frac{1}{i\omega L_m}\right)\hat{V}$$

and find C_m, L_m and R_m in terms of the speaker parameters $M, k, \gamma\,, L, B_0$. C_m , L_m and R_m are called *motional impedances.* To a voltage generator connected to the speaker input terminals, they act just like ordinary capacitors, inductors and resistances but they arise from the electromechanical behavior of the loudspeaker.

Circuit analysis tells us that for any two terminal linear circuit, the voltage and current phasor are related by $\hat{V} = Z(\omega)\hat{I}$ where $Z(\omega)$ is called the *impedance*. The impedance of a resistor, inductor and capacitor are respectively,

$$Z_R = R \qquad Z_L = i\omega L \qquad Z_C = \frac{1}{i\omega C}$$

Impedances connected in parallel add as reciprocals, just like resistances in parallel. Part (c) shows that to a voltage generator connected to the speaker input terminals, the motional impedance appears like a resonant *RLC* circuit,



That's not quite everything. In addition to the motional impedances the loudspeaker coil has ordinary resistance R_C (typically 4-8 Ohms) and ordinary inductance L_C. The full effective circuit looks like,